Math 53 Discussion Problems Sept 3

1. Find the equation of the line tangent to the curve at the given point. Determine whether the curve is concave upwards or downwards at that point.

   (a) $x = 2t^2 + 3, y = t^4, t = -1$

   (b) $x = t + e^t, y = 1 - e^t, t = 0$

2. Find the length of the curves

   (a) $x = \cos t, y = t + \sin t, 0 \leq t \leq \pi$

   (b) $x = \sin(e^t), y = \cos(e^t), 0 \leq t \leq 1$ (Hint: Re-parametrize)

3. Find the area enclosed by the x-axis and the curve $x = t - t^2, y = 1 + e^{-t}$.

4. Consider the circle parametrized by $x = 2 + \cos t, y = \sin t, 0 \leq t \leq 2\pi$. Describe the surfaces obtained by rotating the circle about the x-axis and about the y-axis respectively, then calculate their surface areas.